HSEB Model Question - I (2068)

Mathematics

Grade: XII Full Marks: 100

Time: 3 hrs Pass Marks: 35

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt ALL questions of group A and group B or C.

Group A

1. a) It is required to seat 5 boys and 4 girls in a row so that the girls occupy the even places. How many such arrangements are possible?[2]

b) Prove that: $\frac{1}{1.3} + \frac{1}{2.5} + \frac{1}{3.7} + \frac{1}{4.9} + \ldots = 2\,(1 \text{ ñ} \ln 2)$. [2]

c) Let $a * b = 3a + 2b$ for $a, b \in \mathbb{Z}$. Verify that * is a commutative binary operation on $\mathbb{Z}$. [2]

2. a) Find the equation of a hyperbola in standard position such that the length of transverse axis is 6 and it passes through (4, 2). [2]

b) Find the locus of points which are equidistant from the points (1, 2, 3) and (3, 2, ñ1). [2]

c) Find the cosines of the angle between the vectors:

$\vec{a} = (1, \text{ñ}2, \text{ñ}2)$, $\vec{b} = (2, 1, \text{ñ}2)$. [2]

3. a) Find the derivative of $(\ln x)^{\sinh x}$. [2]

b) Find the integral $\int \frac{dx}{1 + 2\sin x}$. [2]

c) Find the integral $\int \frac{dx}{(x + 7)\sqrt{2 \text{ ñ } x}}$. [2]

4. a) Solve the differential equation: $\frac{dy}{dx} = e^{x+y} + 3x^2 e^y$. [2]

b) From the following data, calculate the expected value of Y when $X = 25$,

	X	Y
Average	5.6	12.5
Standard deviation	3.2	2.4

and correlation coefficient $r = 0.95$. [2]

c) The average percentage of failures in a certain examination is 40. What is the probability that out of 5 candidates, at least 3 will be passed in the examination. [2]

5. a) Show that the number of combinations of n different objects taken r at a time is given by

$$C(n, r) = \frac{n!}{(n \text{ ñ } r)!\, r!} .$$

Also, prove that $C(n, n \text{ ñ } r) = C(n, r)$. [4]

OR

State the multiplication principle of counting. Prove that the number of circular permutations of n different objects taken all at a time is $(n$ ñ $1)!$ [4]

b) What is a group? If a binary operation $*$ is defined on a set

$S = \{a, b, c\}$ by the following Caleyís table

*	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

show that $(S, *)$ is a group. [4]

OR

Let a, b, c and x be elements of a group G. Solve for x if $axb = c$ and $x^2 b = x\, a^{\text{ñ}1}\, c$. [4]

6. a) Find the integral $\int \frac{x}{x^3 + 1}\, dx$. [4]

b) What is a linear differential equation? Solve:

$$(x^2 + 1)\frac{dy}{dx} + 2xy = 3x^2. \quad [4]$$

7. a) State the first mean value theorem of differential calculus and interpret it geometrically. Using it to $f(x) =$ sin x on $[0, x]$, prove that sin $x \le x$ for $x \ge 0$. [4]

b) An urn contains four white, eight black, six red and two green marbles. If three balls are drawn at random, find the probability of getting (i) all white marbles (ii) 2 red and 1 green marbles. [4]

8. a) What is a conic section? Find the equation of the tangent to the parabola $y^2 = 8x$ which is parallel to the straight line $2x$ ñ $3y + 7 = 0$. Also find its point of contact. [4]

b) Define linearly independent vectors. Show that the following vectors are linearly dependent.

$$2\vec{i} + \vec{j} \text{ ñ} \vec{k},\ 3\vec{i} \text{ ñ} 2\vec{j} + \vec{k},\ \vec{i} + 4\vec{j} \text{ ñ} 3\vec{k}. \quad [4]$$

OR

Prove that if θ is the angle between the vectors $\vec{a}$ and $\vec{b}$, then

$$\vec{a}\cdot\vec{b} = a\,b\cos\theta. \quad [4]$$

9. For any positive integer n, prove that

$$(a + x)^n = C(n, 0)a^n + C(n, 1)a^{n\text{ñ}1}\,x + C(n, 2)a^{n\text{ñ}2}\,x^2 + \ldots + C(n, n)\,x^n.$$

Find the term containing x^2, if any, in the expansion of $\left(\frac{2x}{3} \text{ ñ} \frac{3}{2x}\right)^6$. [6]

10. Find the direction cosines of two lines which are connected by the relations $2l + 2m$ ñ $n = 0$, $mn + nl + lm = 0$. [6]

OR

Prove that a plane through three points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0.$$

Also, find the angle between planes $2x$ñ$y+z = 6$ and $x+y+2z = 3$. [6]

11. Lives of two models of refrigerators turned in for new models in a recent survey are

No. of years	No. of refrigerators	
	Model A	Model B
0ñ2	5	2
2ñ4	16	7
4ñ6	13	12
6ñ8	7	19
8ñ10	5	9
10ñ12	4	1

What is the average life of each model of these refrigerators? Which model has more uniformity? [6]

Group B

12. a) Three forces P, Q and R acting on a particle are in equilibrium, the angle between the P and Q is 60 and that between Q and R is 150 . Find the ratios of the forces. [2]

b) A uniform beam, 4 m long, is supported in a horizontal position by two props which are 3 m apart, so that the beam projects one meter beyond one of the props. Show that the force on one of the props is double of that on the other. [2]

c) A pump having a power of 392 W pumps water at the rate of 100 litres per minute. Find the height to which the water is raised. ($g = 9.8$ m/s^2, 1 litre of water = 1 kg). [2]

13 a) A body of weight w is suspended by strings of length 3 m and 4 m attached to two points in the same horizontal line whose distance apart is 5 m. Find the tensions along the strings. [4]

b) A body of mass 49 kg is falling freely under gravity at the rate of 20 m/s. What is the uniform force that will stop it (i) in 2 sec (ii) in 50 cm? ($g = 9.8$ m/s^2.) [4]

OR

A bullet fired into a target losses half its velocity after penetrating 3 cm. How much further will it penetrate? [4]

14. The resultant of two like parallel forces P and Q acting on a rigid body is a force of magnitude $P + Q$ in the same direction as P and Q are. If A and B are any points on the lines of action of P and Q respectively, prove that the resultant divides line segment AB internally in the inverse ratio of the forces. [6]

OR

Define the moment of a force. Forces 1, 2, 4, 5 kg-wts act along the sides of a square taken in order. Prove that their resultant is parallel to a diagonal and find where it cuts the side along which the first force acts.[6]

15 A man travels from *A* to *B* in 45 minutes. At *C*, somewhere between *A* and *B*, it attains its maximum velocity of 45 m per hr. If he travels with uniform acceleration from *A* to *C* and uniform retardation from *C* to *B*, find the distance between *A* and *B*, it being supposed that the man starts from rest at *A* and comes to rest at *B*. [6]

Group C

16 a) Determine graphically the solution set of the following system of inequalities:

$2x + y \geq 2, 3x + 2y \leq 4, \quad x \geq 0, \quad y \geq 0.$ [2]

b) Write a short note on accuracy of a numerical method. [2]

c) Apply the Simpons's rule to approximate the value of $\int_1^4 e^x \ln x \, dx$ with $n = 3$. [2]

17 a) Using the simplex method, maximize $p = 6x$ ñ $9y$ subject to

$2x$ ñ $3y \leq 6, \quad x + y \leq 20, \quad x \geq 0, \quad y \geq 0.$ [4]

b) Use Bisection method to find solution accurate to within 10 $^{\text{ñ}2}$ for x^3 ó $7x^2 + 14x$ ó $6 = 0$ on the interval [1, 3.2]. [4]

OR

Write three methods for measuring error. Approximate $\sqrt{11}$ by Newton-Raphson's method with accuracy 0.00001. [4]

18. Find the approximate solution of the following system of equations by matrix inversion method:

$2x$ ñ $y + z =$ ñ2, $x + y$ ñ $2z =$ ñ9, $z + 2y + z = 9$. [6]

19. Derive the trapezoidal rule. The capacity of a battery is a measure of $\int i \, dt$, where i is the current. Estimate, using the Trapezium rule, the capacity of a battery whose current was measured over an eight hour period with the results shown below

Time/hours	0	1	2	3	4	5	6	7	8
Current/Amps	25.2	29.0	31.8	36.5	33.7	31.2	29.6	27.3	28.6

[6]

OR

Compute an approximate value of $\int_0^1 (1 + x^2)^{1} \, dx$ by using the composite trapezoid rule with three points. Then compare with the actual value of the integral. Next, determine the error formula and numerically verify an upper bound on it. [6]

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