HSEB Model Question - II (2068)

Mathematics

Grade: XII | Full Marks: 100

Time: 3 hrs | Pass Marks: 35

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt ALL questions of group A and group B or C.

Group A

1. a) In an examination paper containing 10 questions, a candidate has to answer 7 questions. If two questions are made compulsory, in how many ways can he choose 7 questions in all? [2]

b) Find the middle term in the expansion of $\left(2x + \frac{1}{3x^2}\right)^9$. [2]

c) Let $S = \{\text{ñ}1, 1\}$ and $*$ denote the usual operation of multiplication. Represent it by Cayleyís table. Show that $*$ is a binary operation on S.[2]

2. a) Find the eccentricity and the foci of the ellipse:

$$x^2 + 4y^2 \text{ ñ } 4x + 24y + 24 = 0.$$ [2]

b) Find the point where the line through the points (1, 2, 3) and (4, ñ 4, 9) meets the zx-plane. [2]

c) Are the three points with position vectors $\vec{i} + 2\vec{j} + 4\vec{k}$, $2\vec{i} + 5\vec{j} - \vec{k}$ and $3\vec{i} + 8\vec{j} - 6\vec{k}$ collinear? Justify your answer. [2]

3. a) Using L'Hospitalís rule, evaluate

$$\lim_{x\to 0} \frac{e^x + e^{\text{ñ}x} \text{ ñ } 2\cos x}{\sin^2 x}.$$ [2]

b) Evaluate:

$$\int \frac{dx}{\sqrt{(x \text{ ñ } \alpha)(x \text{ ñ } \beta)}} \ (\beta > \alpha).$$ [2]

c) If $\vec{a} = 6\vec{i} + 3\vec{j} \text{ ñ } 5\vec{k}$ and $\vec{b} = \vec{i} \text{ ñ } 4\vec{j} + 2\vec{k}$ show that $\vec{a}$ $\vec{b}$ is perpendicular to $\vec{a}$. [2]

4. a) Solve:

$$x\frac{dy}{dx} + y \text{ ñ } 1 = 0.$$ [2]

b) If $n = 10$, $\Sigma X = 60$, $\Sigma Y = 60$, $\Sigma X^2 = 400$, $\Sigma Y^2 = 580$ and $\Sigma XY = 415$, find the correlation coefficient between the two variables. [2]

c) Two dice are rolled once. What is the probability of getting a total of 9 or 6? [2]

5. a) In how many ways can the letters of the word ìCOMPUTERî be arranged so that

i. all the vowels are always together?

ii. the vowels may occupy only odd positions? [4]

b) Given the algebraic structure $(G,*)$ with $G = \{1, \omega, \omega^2\}$ where ω represents an imaginary cube root of unity and $*$ stands for the binary operation of multiplication, show that $(G,*)$ is a group. [4]

6. a) Find the equation of the tangent to the parabola $y^2 = 4ax$ at the point (x_1, y_1). Express it in the slope form.

OR

What is a conic section? Find the equation of the parabola in the standard form. [4]

b) Find the equation of the plane through the point (2, 1, 4) and perpendicular to each of the planes

$$9x - 7y + 6z + 48 = 0 \text{ and } x + y + z = 0.$$ [4]

7. a) Evaluate:

$$\int \frac{dx}{a + b\cos x} \ (a > b > 0).$$ [4]

b) Solve: $x^2 \frac{dy}{dx} + y^2 = xy$

OR

Solve: $(1 ñ x^2) \frac{dy}{dx}$ ñ xy = 1 [4]

8. a) Find Karl Pearsonís coefficient of skewness from the following distribution. [4]

Marks	Above 20	Above 30	Above 40	Above 50	Above 60
No. of students	50	46	30	24	8

b) The chance that A can solve a certain problem is $\frac{1}{4}$ and the chance that B can solve it is $\frac{2}{3}$. Find the chance that (i) the problem will be solved if they both try (ii) A solves but B cannot.

OR

Suppose that in a certain city 60% of all the recorded births are male. Suppose we select 5 birth records from population. What is the probability that

i. exactly three of them are male?

ii. 4 or more are male? [4]

9. Show that

$$\sum_{n=1}^{\infty} \frac{n^2}{(n+1)^2} = e ñ 1.$$ [6]

10. Define scalar product of two vectors. Find the geometrical interpretation of scalar product of two vectors. Prove vectorically that

$\cos(A + B) = \cos A \cos B ñ \sin A \sin B.$ [6]

11. State Rolleís theorem. Interpret it geometrically. Verify Rolleís theorem for the function

$$f(x) = x(x - 1)^2 \text{ in } [0,1]$$

Also, find the point on the curve where the tangent is parallel to the x-axis. [6]

OR

Find from first principle the derivative of $\ln \cos^{ñ1} x$. [6]

Group ëBí

12. a) Forces equal to 7P, 5P and 8P acting on a particle are in equilibrium. Find the angle between the latter pair of forces. [2]

b) A body is projected vertically upwards with a velocity of 19.6 m/s. How long will it take to reach a point 294m below the point of projection? (g = 9.8m/s) [2]

c) A body of mass 50 kg falling from a certain height is brought to rest after striking the ground with a speed of 5 m/s. If the resistance force of the ground is 500N, find the duration of the contact. [2]

13. a) P and Q are two like parallel forces acting at A and B. Show that if they interchange positions, the point of application of the resultant is displaced by a distance $\frac{P ñ Q}{P + Q} \cdot AB$.

OR

Forces 1N, 2N and 3N act at a point in direction parallel to the sides of an equilateral triangle taken in order. Find their resultant. [4]

b) Prove that the sum of the kinetic and potential energies of a freely falling body remains constant throughout the motion. [4]

14. The horizontal and the vertical components of the initial velocity of a projectile are U and V respectively. If R be the horizontal range and H, the greatest height attained, prove that

i) $\frac{4H}{R} = \frac{V}{U}$ ii) $\left(\frac{R}{U}\right)^2 = \frac{8H}{g}$.

OR

A cat seeing a mouse at a distance of 15m before it, starts from rest with an acceleration of 2 m/s^2 and pursues it. If the mouse be moving uniformly with a velocity of 14 m/s, find when and where the cat will catch the mouse. [6]

15. Define the moment of a force about a point and interpret its geometrical meaning. Prove that the algebraic sum of the moments of two intersecting forces about any point in their plane is equal to the moment of their resultant about the same point. [6]

Group ëCí

16. a) If a man rides his car at 25 km/hr, he has to spend Rs.2 per km on petrol. If he rides it at a faster speed of 40 km/hr, the petrol cost increases to Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Formulate the above problem as a linear programming problem. [2]

b) Convert the decimal number 2011 into octal form. [2]

c) Is the following equations diagonally dominant:

$12x + 3y$ ñ $5z = 1, x + 5y + 3z = 28, 3x + 7y + 13z = 1$? [2]

17. a) Using Gauss elimination method, solve the following system of equations: [4]

$x + 3y - z = -2$

$3x + 2y - z = 3$

$-6x - 4y - 2z = 18.$

OR

Solve the following equations using Gauss-Seidal method:

$2x_1 - x_2 = 8$

$3x_1 + 7x_2 = -5$

b) Evaluate the following integral using Simpsonís rule:

$\int_0^1 \frac{dx}{1+x^2}$, taking 4 equal intervals (i.e. $n = 4$) [4]

18. Using Simplex method, maximize $Z = 5x_1 + 7x_2$ subject to:

$2x_1 + 3x_2 \le 13$

$3x_1 + 2x_2 \le 12$

$x_1, x_2 \ge 0.$ [6]

19. Show that the equation $f(x) = x^3 - 18 = 0$ has only one positive root. Using bisection method, find the positive root correct to 3 places of decimal in the interval (2, 3). [6]

OR

Use Newton-Raphson method to find the positive root of $x^3 + 3x$ ñ $5 = 0$ lying between 1 and 2 correct to three places of decimals. [6]

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